



# One-loop electroweak corrections to $e^+e^-$ into three-jets

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We describe the impact of the full one-loop Electro-Weak terms of  $\mathcal{O}(\alpha_s\alpha_{\text{EM}}^3)$  entering the electron-positron into three-jet cross-section. We include both factorisable and non-factorisable virtual corrections, photon bremsstrahlung but not the real emission of  $W^\pm$  and  $Z$  bosons. We show preliminary results and we discuss the impact of the Electro-Weak corrections on three-jet observables.

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## 1. Three-jet Events at Leptonic Colliders

Strong (QCD) and Electro-Weak (EW) interactions are two fundamental forces of Nature, the latter in turn unifying Electro-Magnetic (EM) and Weak interactions in the Standard Model (SM). A clear hierarchy exists between the strength of these two interactions at the energy scales probed by past and present high energy particle accelerators (e.g., LEP, SLC, HERA and Tevatron): QCD forces are stronger than EW ones. This argument, however, is only valid in lowest order in perturbation theory.

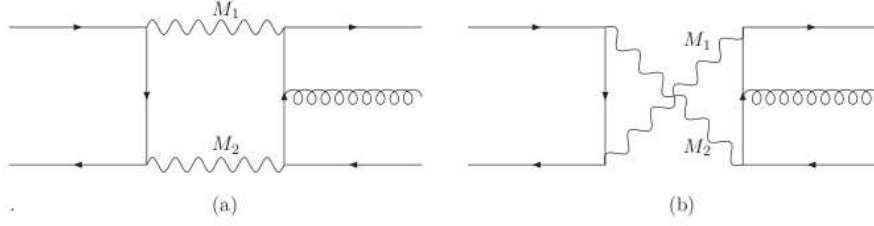
A peculiar feature in fact distinguishing QCD and EW effects in higher orders is that the latter are enhanced by double logarithmic factors,  $\log^2(\frac{s}{M_W^2})$ , which, unlike in the former, do not cancel for ‘infrared-safe’ observables [1, 2, 3]. The origin of these ‘double logs’ is well understood. It is due to a lack of the Kinoshita-Lee-Nauenberg (KLN) [4] type cancellations of Infra-Red (IR) – both soft and collinear – virtual and real emission in higher order contributions originating from  $W^\pm$  (and, possibly,  $Z$ ) exchange. This is in turn a consequence of the violation of the Bloch-Nordsieck theorem [5] in non-Abelian theories [6]. The problem is in principle present also in QCD. In practice, however, it has no observable consequences, because of the final averaging of the colour degrees of freedom of partons, forced by their confinement into colourless hadrons. This does not occur in the EW case, where the initial state has a non-Abelian charge, dictated by the given collider beam configuration, such as in  $e^+e^-$  collisions.

These logarithmic corrections are finite (unlike in QCD), as the masses of the weak gauge bosons provide a physical cut-off for  $W^\pm$  and  $Z$  emission. Hence, for typical experimental resolutions, softly and collinearly emitted weak bosons need not be included in the production cross-section and one can restrict oneself to the calculation of weak effects originating from virtual corrections and affecting a purely hadronic final state. Besides, these contributions can be isolated in a gauge-invariant manner from EM effects [3], at least in some specific cases, and therefore may or may not be included in the calculation, depending on the observable being studied. As for purely EM effects, since the (infinite) IR real photon emission cannot be resolved experimentally, this ought to be combined with the (also infinite) virtual one, through the same order, to recover a finite result, which is however not doubly logarithmically enhanced (as QED is an Abelian theory).

In view of all this, our aim is the computation of the full one-loop EW effects entering three-jet production in electron-positron annihilation at any collider energy via the subprocesses  $e^+e^- \rightarrow \gamma^*, Z \rightarrow \bar{q}qg$ <sup>1</sup>. Ref. [9] tackled part of these, in fact, limitedly to the case of  $W^\pm$  and  $Z$  (but not  $\gamma$ ) exchange and when the higher order effects arise only from initial or final state interactions (these represent the so-called ‘factorisable’ corrections, i.e., those involving loops not connecting the initial leptons to the final quarks, which are the dominant ones at  $\sqrt{s} = M_Z$ , where the width of the  $Z$  resonance provides a natural cut-off for off-shellness effects). The remainder, ‘non-factorisable’ corrections, while being typically small at  $\sqrt{s} = M_Z$ , are expected to play a quantitatively relevant role as  $\sqrt{s}$  grows larger. By studying the full set of the one-loop EW corrections, we improve on the results of Ref. [9] in two respects: (i) we include now all the non-factorisable terms; (ii) we also incorporate previously neglected genuine QED corrections, including photon bremsstrahlung.

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<sup>1</sup> See Ref. [7] for the corresponding one-loop corrections to the Born process  $e^+e^- \rightarrow \bar{q}q$  and Ref. [8] for the  $\sim n_f$  component of those to  $e^+e^- \rightarrow \bar{q}qgg$  (where  $n_f$  represents the number of light flavours).



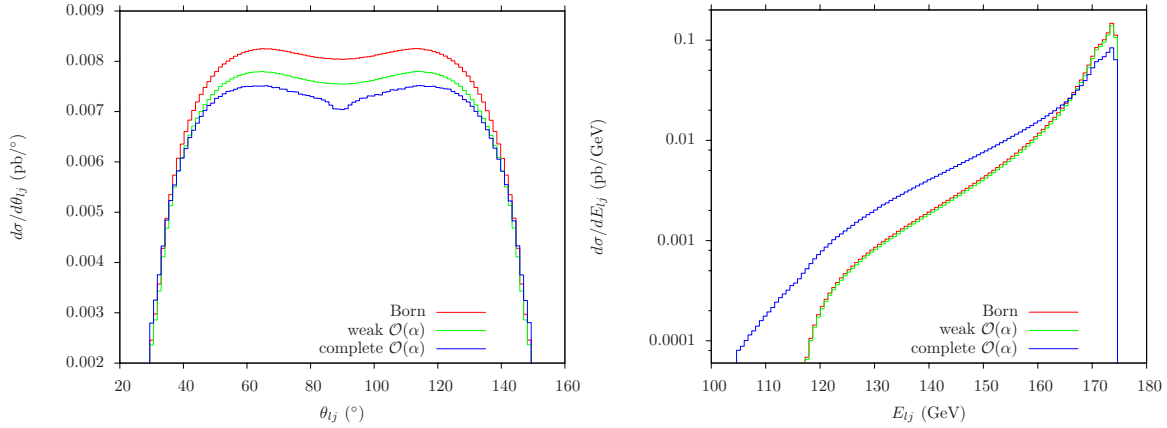
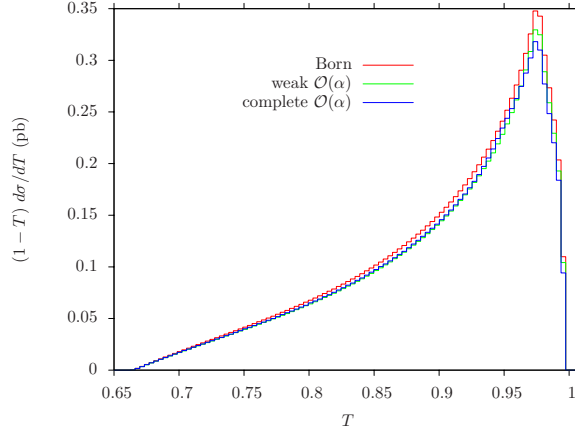
**Figure 1:** Pentagon graphs. The gauge bosons in the loop can be  $W$ ,  $Z$  or  $\gamma$ .

Combining the logarithmic enhancement associated to the genuinely weak component of the EW corrections to the fact that  $\alpha_s$  steadily decreases with energy, unlike  $\alpha_{\text{EM}}/\sin^2 \theta_W$ , in general, one expects one-loop EW effects to become comparable to QCD ones at future Linear Colliders (LCs) [10] running at TeV energy scales<sup>2</sup>. In contrast, at the  $Z$  mass peak, where logarithmic enhancements are not effective, one-loop EW corrections are expected to appear at the percent level, hence being of limited relevance at LEP1 and SLC, where the final error on  $\alpha_s$  is of the same order or larger [11], but of crucial importance at a GigaZ stage of a future LC [9], where the relative accuracy of  $\alpha_s$  measurements is expected to be at the 0.1% level or smaller [12]. On the subject of higher order QCD effects, it should be mentioned here that a great deal of effort has recently been devoted to evaluate two-loop contributions to the three-jet process [13] while the one-loop QCD results have been known for quite some time [14].

As intimated, in the case of  $e^+e^-$  annihilations, the most important QCD quantity to be extracted from multi-jet events is  $\alpha_s$ . The confrontation of the measured value of the strong coupling constant with that predicted by the theory through the renormalisation group evolution is an important test of the SM or else an indication of new physics, when its typical mass scale is larger than the collider energy, so that the new particles cannot be produced as ‘real’ detectable states but may manifest themselves through ‘virtual’ effects. Not only jet rates, but also jet shape observables would be affected. Our calculation involves the full one-loop EW corrections to three-jet observables in electron-positron annihilations, including also non-factorisable corrections such as the ones generated via the interference of the pentagon graphs in Fig. 1 with the tree-level ones. Hence, our calculation not only accounts for the mentioned double logarithms, but also all single ones as well as the finite terms arising through the complete  $\mathcal{O}(\alpha_s \alpha_{\text{EW}}^3)$ . We account for all possible flavours of (anti)quarks in the final state, with the exception of the top quark. The latter however appears in some of the loops whenever a  $b\bar{b}g$  final state is considered, in particular notice that, in this case, we will also have to include loops involving the Higgs boson coupling to (anti)top quark lines.

We expect that all such corrections are of a few percent at  $\sqrt{s} = M_Z$  and that they grow to a few tens of percent at LC energies. Hence, while their impact is not dramatic in the context of LEP1 and SLC physics at a GigaZ stage of future LCs they ought to be taken into account in the experimental fits. Even more so, it is the case of future LCs running at and beyond the TeV range.

<sup>2</sup>For example, at one-loop level, in the case of the inclusive cross-section of  $e^+e^-$  into hadrons, the QCD corrections are of  $\mathcal{O}(\frac{\alpha_s}{\pi})$ , whereas the EW ones are of  $\mathcal{O}(\frac{\alpha_{\text{EW}}}{4\pi} \log^2 \frac{s}{M_W^2})$ , where  $s$  is the collider CM energy squared, so that at  $\sqrt{s} \approx 1.5$  TeV the former are identical to the latter, of order 9% or so.

**Figure 2:** Distributions of the leading jet angle and energy**Figure 3:** Thrust distribution

## 2. Calculation and preliminary results

In this section we sketch the main features of the calculation and we present preliminary results. The detailed description of the calculation and a wider phenomenological study appeared elsewhere [15].

Since with respect to Refs. [9] we include QED corrections, loop diagrams can contain one or two photons and give rise to infrared (IR) and collinear divergences. We regularise the divergences by simply inserting a mass  $\lambda$  for the photon and a mass  $m_f$  for all fermions. This is also done in the case of the bremsstrahlung contribution before integrating over the phase space for the emitted photon. In order to check the cancellation of the IR divergences between real and virtual corrections, we successfully verified the independence of their sum against variation of the photon mass  $\lambda$ . Another key feature of this calculation is the occurrence of pentagon graphs, as shown in Fig. 1. Such graphs involve five-point Passarino-Veltman (PV) [16] functions with up to three powers of momenta in the numerator. We have handled these in two separate ways (with two inde-

pendent codes), in order to check for possible numerical instabilities. In the first case the integrals are simply evaluated using routines in LoopTools v2.2 [17]<sup>3</sup>. In the other we use the standard PV reduction, carried out exhaustively until only scalar pentagon integrals appear. The latter are available in the library FF1.9 [19]. A comparison of the numerical results provided by the two codes yielded satisfactory agreement between the two methods. Also the squared amplitudes for the real emission process have been evaluated by using two independent tools (ALPHA [20] and MadGraph [21]) finding perfect agreement. The integration over the three- and four-body phase space is performed numerically by means of a Monte Carlo method, using standard importance sampling techniques for the variance reduction.

In order to perform a preliminary analysis, we considered an  $e^+e^-$  collider at  $\sqrt{s} = 350$  GeV and we used a realistic experimental setup: partonic momenta are clustered into jets according to the Durham jet algorithm [22] (e.g. when  $y_{ij} < y_{min}$  with  $y_{min} = 0.001$ ), the jets are required to lie in the central detector region  $30^\circ < \theta_{jets} < 150^\circ$  and we require that the invariant mass of the jet system is larger than  $0.75 \times \sqrt{s}$ . If a real photon is present in the final state, it is clustered according to the same algorithm, but we require that at least 3 “hadronic” jets are left at the end. Finally, we sum over the final-state quarks.

In Figs. 2 and 3, some examples of the impact of the full EW corrections on three-jet observables are shown. The distributions are plotted in Born approximation (red line), including the complete 1-loop EW corrections (blue) and including only the weak corrections (green). In Fig. 2 the distributions of the angle (left) of the most energetic (leading) jet and the energy (right) of the leading jet are shown. In Fig. 3 the distribution of the thrust shape variable is plotted.

Even in this limited set of observables, the impact of the EW corrections is evident. The leading jet angle and energy distributions show clearly the effect of the purely weak and QED corrections. The 5-6% effect on the thrust distribution indicates that the EW corrections are unavoidable to carry out a precise measurement of  $\alpha_s$  at a future LC with a 0.1% accuracy. A more complete set of observables are discussed in detail in the paper [15].

Before concluding, it is worth noticing that this calculation can be used as a starting point (by exploiting the crossing symmetry) to calculate the complete 1-loop EW corrections to  $\gamma^*/Z$  jet production at hadron colliders, which is a process of great interest for physics at the forthcoming LHC. This study is under consideration.

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<sup>3</sup>We implemented also directly in an independent fortran routine the expressions for the five point functions of Ref. [18], finding agreement with LoopTools.

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